MULTIMEDIA UNIVERSITY®

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2022/2023

TMA1201 - DISCRETE STRUCTURES AND PROBABILITY

(TC1L / TT1L)

05 OCTOBER 2023
9.00 a.m. - 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of 7 pages (inclusive of the cover page).
2. Answer ALL questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Please write all your answers (including all steps) in the Answer Booklet provided.
4. Key formulas and statistical tables are provided at the Appendix pages.

Question 1 (25 marks)

a) Two relations A and B defined on set $W = \{\ 0 < w < 10 \wedge w \in$ *integers that divides* $24\ \}$ are given as below:

$A = \{\ (2, 1)\ ,\ (4, 1)\ ,\ (4, 6)\ ,\ (6, 1)\ ,\ (6, 4)\ ,\ (6, 6)\ \}$
$B = \{\ (1, 6)\ ,\ (2, 1)\ ,\ (2, 4)\ ,\ (4, 6)\ \}$

(i) State the elements of the relation $A - B$. [1 mark]

(ii) Determine if the relation stated in (i) is reflexive, symmetric, antisymmetric and/or transitive. Explain your answers and provide a counterexample wherever required. [6 marks]

(iii) Find set B^3. [$1\frac{1}{2}$ marks]

b) Given two predicates as below.

$D(x)$: x is a digital book
$L(x)$: x can be borrowed from the library

(i) Translate each of the following sentences into predicate statements using logical connectives and quantifiers, wherever required.

All non-digital books can be borrowed from the library.
The book *Learn Logic* cannot be borrowed from the library.
Thus, *Learn Logic* is a digital book that cannot be borrowed from the library.

[3 marks]

(ii) Apply formal reasoning using the Rules of Inference to prove the arguments in (i). Let the Universe of Discourse (UOD) be all books in the world.

[5 marks]

c) Given a propositional function, $P(x, y) : x + \frac{1}{x} = y$, where $x, y \in \mathbb{Z}$. Determine the truth value of the following statement. Explain your answers and provide a counterexample wherever required.

(i) $P(1, 2)$ [1 mark]

(ii) $\exists x P(x, 0)$ [$1\frac{1}{2}$ marks]

(iii) $\forall x \exists y P(x, y)$ [$1\frac{1}{2}$ marks]

Continued...

d) Given a snippet of codes as shown below.

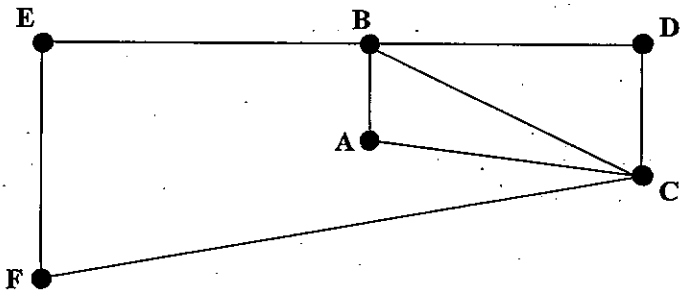
Line No.	Description	Processing Time
1	if $n = 0 \mid n = 1$	
2	return 1	
3	else for $i = 1$ to n	
4	func(n) = func($n-1$) + func($n-2$)	

(i) Complete the table with the processing time at each line of code. [3 marks]

(ii) State the theta notation, Θ for this portion of codes. [$1\frac{1}{2}$ marks]

Question 2 (25 marks)

a) Given a graph as below.



(i) Write the adjacency matrix for the graph. [3 marks]

(ii) Determine if the graph has a Euler Circuit and/or Euler Trail. Explain your answers and provide a sample Euler Circuit and/or Euler Trail, if any. [2 marks]

(iii) Create a spanning tree using the Breadth-First Search (BFS) algorithm with **A** as the root. Note that alphabetical ordering applies when selecting a node. [6 marks]

Continued...

b) Given the state-transition table for a finite-state automaton (FSA) below.

State	Next State	
	a	***b***
S_0	S_2	S_1
S_1	S_1	S_1
S_2	S_2	S_2

(i) Construct the state-transition diagram for the FSA. [3 marks]

(ii) Give an example of a 6-bits string that will be accepted by the FSA. [1 mark]

(iii) State the language recognises by the FSA. [1 mark]

c) Find the number of ways to arrange the letters of the word HOUSE if,

(i) the vowels must be together? [1 mark]

(ii) there are no restrictions? [1 mark]

(iii) the word is in its plural form, HOUSES. [1 mark]

d) Given that 25% of residents at a Taman choose jogging as their favourite exercise. In a sample of 10 residents, find the probability that at most 8 residents do NOT choose jogging as their favourite exercise. Write your answers in 4 decimal places. [2 marks]

e) Blood pressure readings are assumed to be normally distributed with a mean systolic blood pressure of 120 and variance of 64.

(i) Find the probability that for any patient, the blood pressure reading will be between 100 and 130, inclusive. [2 marks]

(ii) Find the expected number of people who may have blood pressure readings of more than 135 in a sample of 200 people. [2 marks]

End of Questions.

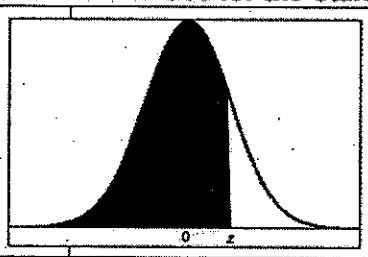
APPENDIX

Name	Tautological Arguments	Premises	Conclusion
Addition	$p \Rightarrow p \vee q$	1. p	$p \vee q$
Simplification	$(p \wedge q) \Rightarrow p$	1. p 2. q	p
Modus Ponens	$[p \wedge (p \rightarrow q)] \Rightarrow q$	1. p 2. $p \rightarrow q$	q
Modus Tollens	$[\neg q \wedge (p \rightarrow q)] \Rightarrow \neg p$	1. $\neg q$ 2. $p \rightarrow q$	$\neg p$
Hypothetical Syllogism	$[(p \rightarrow q) \wedge (q \rightarrow r)] \Rightarrow (p \rightarrow r)$	1. $p \rightarrow q$ 2. $q \rightarrow r$	$p \rightarrow r$
Disjunctive Syllogism	$[(p \vee q) \wedge \neg p] \Rightarrow q$	1. $p \vee q$ 2. $\neg p$	q
Resolution	$[(p \vee q) \wedge (\neg p \vee r)] \Rightarrow (q \vee r)$	1. $p \vee q$ 2. $\neg p \vee r$	$q \vee r$
Conjunction	$p \wedge q \Rightarrow (p \wedge q)$	1. p 2. q	$p \wedge q$

Identity laws $p \wedge T \Leftrightarrow p$ $p \vee F \Leftrightarrow p$	**Associative laws** $(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$ $(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$	**Conversion of Implication** $p \rightarrow q \Leftrightarrow \neg p \vee q$ $p \vee q \Leftrightarrow \neg p \rightarrow q$
Domination laws $p \vee T \Leftrightarrow T$ $p \wedge F \Leftrightarrow F$	**Absorption Laws** $p \vee (p \wedge q) \Leftrightarrow p$ $p \wedge (p \vee q) \Leftrightarrow p$	**Contrapositive** $p \rightarrow q \Leftrightarrow \neg q \rightarrow \neg p$ $p \leftrightarrow q \Leftrightarrow \neg q \leftrightarrow \neg p$
Idempotent laws $p \vee p \Leftrightarrow p$ $p \wedge p \Leftrightarrow p$	**Commutative laws** $p \vee q \Leftrightarrow q \vee p$ $p \wedge q \Leftrightarrow q \wedge p$	**Conversion of Equivalence** $p \leftrightarrow q \Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p)$
Negation laws $p \vee \neg p \Leftrightarrow T$ $p \wedge \neg p \Leftrightarrow F$ **Double Negation laws** $\neg(\neg p) \Leftrightarrow p$	**Distributive laws** $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$ $(p \vee q) \wedge r \Leftrightarrow (p \wedge r) \vee (q \wedge r)$ $(p \wedge q) \vee r \Leftrightarrow (p \vee r) \wedge (q \vee r)$	**De Morgan's laws** $\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$ $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$

STANDARD NORMAL TABLE

Cumulative Probabilities for the Standard Normal (Z) Distribution



Values in the table correspond to the area under the curve of a standard normal random variable for a value at or below z.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5676	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998									
4.0	0.99997									
4.5	0.999997									
5.0	0.9999997									

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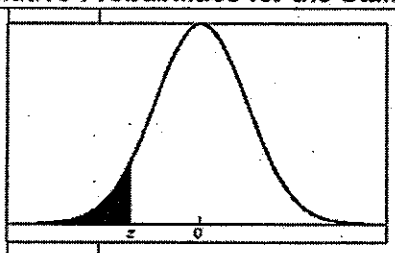

Values in the table correspond to the area under the curve of a standard normal random variable for a value at or below z.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-5.0	0.0000003									
-4.5	0.000003									
-4.0	0.00003									
-3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

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